

Interaction Analysis and Decomposition Principle for Control Structure Design of Large-scale Systems^{*}

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Abstract Industrial processes are mostly large-scale systems with high order. They use fully centralized control strategy, the parameters of which are difficult to tune. In the design of large-scale systems, the decomposition according to the interaction between input and output variables is the first step and the basis for the selection of control structure. In this paper, the decomposition principle of processes in large-scale systems is proposed for the design of control structure. A new variable pairing method is presented, considering the steady-state information and dynamic response of large-scale system. By selecting threshold values, the related matrix can be transformed into the adjoining matrixes, which directly measure the couple among different loops. The optimal number of controllers can be obtained after decomposing the large-scale system. A practical example is used to demonstrate the validity and feasibility of the proposed interaction decomposition principle in process large-scale systems.

Keywords control structure, interaction analysis, interaction decomposition, threshold value

1 INTRODUCTION

In practical chemical processes, many alternatives are usually available, ranging from a fully centralized system to a fully decentralized one. If the fully centralized control method is adopted, the burden of calculation will be aggravated because of the high dimension of the process [1]. The internal interaction in the system is ignored, and it is difficult to tune the parameters [2, 3]. However, for the fully decentralized control system, which includes many single variable proportion-integral-derivative (PID) controllers, it is difficult to achieve a reasonable control performance because of the strong interactions among control loops. To solve this problem, the block decentralized control strategy, which lies between these two extreme control structures, is required. Decomposition, considering the interaction among the control loops, can provide the criterion to control structure selection. Thus research on the interaction analysis and interaction decomposition of large-scale systems is necessary.

The relative gain array (RGA) [4] based techniques has many important advantages, such as simplicity of calculation, as it only uses the steady-state gain matrix of the process. However, using steady-state gain alone may result in erroneous interaction measures and loop-pairing decisions. To overcome the limitations of RGA based on the loop-pairing criterion, several pairing methods have been proposed by considering both steady-state and transient information of the process. For the dynamic relative gain array (DRGA) method [5], a proportional output optimal controller is designed based on the state space approach, and the resulting controller gain matrix is used to define the matrix using the available dynamic process model. However, DRGA is often controller dependent, making it more

difficult to calculate and to be understood by practical control engineers. To combine the advantages of both RGA and DRGA, an effective relative energy array (ERGA) [6] based loop-pairing criterion is defined by employing the steady-state gain and bandwidth of the process transfer function element in the frequency domain. Although ERGA considers both steady-state and transient information of the process, its calculation depends on the bandwidth selection of the process without considering time delay. To address this disadvantage of ERGA, He *et al.* [7] introduced a relative normalized gain array (RNGA) based loop-pairing criterion by considering the steady-state gain and average residence time (AST) of the process in the time domain. As RNGA considers both steady-state gain and dynamic response, it provides a more comprehensive description of loop interactions. However, AST only focuses on the sum of the time constant and the time delay and it never considers the influence of time delay when the average residence time is constant. Ren *et al.* [8] introduced the relative energy gain array (REGA) method, which can balance the relationship between time constant and time delay. REGA is controller independent and is more superior to other existing loop-pairing methods. It will be introduced in Section 3.

Some researchers have conducted studies on loop decomposition-based interaction analysis. Zhu and Jutan [9] presented a method to decompose a system into several single-input single-output control loops with relative interactions. Similarly, Cai *et al.* [10] determined the gain and phase changes of a transfer function with other loops closed by employing the concept of integrated error. Based on the equivalent transfer function, the parameters of a stable, proper, and causal ideal diagonal decoupler can be easily

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determined by proper parameter assignments of the decoupled transfer function matrix elements. Although these two methods can be effectively used in a two-dimensional system, they are difficult to apply in high-dimension systems. To solve the problem of decomposing high-dimension systems, some new interaction decomposition methods have been published. Mario and Arthur [11] proposed a Gramian-based measure of the interaction in multivariable plants, which provides support for decentralized input-output pairing and for a rich controller architecture selection in continuous and discrete-time framework. Nevertheless, the decomposition result is not uniquely determined. Shen *et al.* [12] analyzed RGA [7] to obtain an equivalent transfer function to decompose the multiple inputs and multiple outputs (MIMO) process control system and proposed a sparse control structure to provide a compromise between system performance and structural complexity. However, the sparse control structure is very complex. By defining a relative control performance index, RGA can be transferred to a relative gain array that can be decomposed [13]. A C—H diagram is proposed to measure the interaction of control loops and to facilitate the control structure selection process. The RGA based loop-pairing criterion ignores the transient information of process, and it may lead to incorrect pairing results. Drawing the C—H diagram of high-dimension systems is very complicated, and the method can not achieve the unique interaction decomposition.

Although some of these proposed techniques may satisfy control structure selection, no systematic method is available for control structure selection. Thus this paper intends to examine interaction analysis and interaction decomposition in large-scale system to design control systems. A new interaction analysis method based on the transfer function matrix of open loop, REGA, is introduced, which considers both the steady-state gain and the transient information of the process. REGA is then transferred into adjoining matrixes by selecting different threshold values. As the number of the threshold value increases, the number of optimal controller, N_{opt} , will be determined after decomposing the large-scale system. Two examples are presented to demonstrate that the proposed method is simple, effective, and easy to be implemented.

2 PRELIMINARIES

Considering an $n \times n$ system with multivariable feedback control structure as shown in Fig. 1, where $\mathbf{G}(s) = [g_{ij}(s)]_{n \times n}$ is the transfer function matrix of system; $\mathbf{C}(s)$ is the transfer function matrix of multivariable controller to be selected from all types of controllers ranging from full decentralized controllers to centralized controllers including block diagonal controllers; and $\mathbf{r}$, $\mathbf{u}$ and $\mathbf{y}$ are vectors of references, manipulated and controlled variables respectively.

The relative gain for the variable pairing $y_i - u_j$ is defined as the ratio of two gains representing the

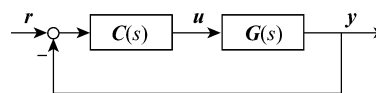


Figure 1 Block diagram of general multivariable control system

process gain in an isolated loop and the apparent process gain in the same loop when all others loops

are closed, $\lambda_{ij} = \frac{(\partial y_i / \partial u_j)_{u_{i \neq j} \text{ constant}}}{(\partial y_i / \partial u_j)_{y_{k \neq i} \text{ constant}}} = g_{ij} [\mathbf{G}^{-1}]_{ji}$.

RGA, $\Lambda(\mathbf{G})$, in matrix form, is defined as, $\Lambda(\mathbf{G}) = \{\lambda_{ij} | i, j = 1, 2, \dots, n\} = \mathbf{G} \otimes \mathbf{G}^{-T}$, where $\otimes$ is the Hadamard product and $\mathbf{G}^{-T}$ is the transpose of the inverse of $\mathbf{G}$.

One of the main advantages of this pairing rule is that the interaction evaluation depends on the steady-state gains only. However, this pairing method may result in wrong interaction measures and loop-pairing decisions. To overcome the limitation of RGA based loop pairing criterion, the loop pairing methods combining with steady state information and transient character are developed. The ERGA considers the steady state gain and bandwidth of the process transfer function element. However, since the calculation of ERGA depends on the critical frequency point of individual element, different section criteria for critical frequency points will result in different ERGAs. To improve the ERGA, the RNGA based loop-pairing criterion can consider both steady-state and transient information in the time domain. RNGA, Φ , in matrix form, is defined as $\Phi = [\varphi_{ij}]_{n \times n} = \mathbf{K}_N \otimes \mathbf{K}_N^{-T}$, where $\mathbf{K}_N = [k_{N,ij}]_{n \times n}$ with $k_{N,ij} = g_{ij}(j0) / \tau_{ar,ij}$, where $\tau_{ar,ij}$ is the average residence time which is the sum of the time constant and time delay. It never considers the effect of time delay for variable pairing when the average residence time is constant. The following example is provided as an illustration.

Example 1 Consider a 2×2 process with the transfer function matrix [8]:

$$\mathbf{G}(s) = \begin{bmatrix} \frac{7e^{-3s}}{s^2 + 5s + 1} & \frac{7e^{-5s}}{s^2 + 3s + 1} \\ \frac{16e^{-8s}}{s^2 + 3s + 1} & \frac{-16e^{-3s}}{s^2 + 8s + 1} \end{bmatrix}.$$

The steady-state RGA and RNGA are obtained as

$$\mathbf{G}_{\text{RGA}} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}, \quad \mathbf{G}_{\text{ERGA}} = \begin{bmatrix} 0.1589 & 0.8411 \\ 0.8411 & 0.1589 \end{bmatrix}, \quad \text{and} \\ \mathbf{G}_{\text{RNGA}} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}.$$

The RGA and RGA matrixes can not determine the pairing result. RGA uses the steady-state gain alone and can not consider the dynamic property of the process. Although RGA considers both the steady-state and dynamic information of the process, it never takes time delay into account. Thus a reasonable variable pairing can be obtained only if the time constant and time delay can be perfectly balanced.

3 INTERACTION ANALYSIS FOR LARGE-SCALE SYSTEMS

Variable pairing is an important step in the design of the process control system [14]. The time delay is always present in practical systems, and its level plays an important role in control performance and variable pairing. This paper presents a variable pairing method, REGA, which can balance the relationship between the time delay and the time constant.

3.1 The index of energy consumption

Suppose that a system dynamic model is available and that the transfer function matrix of the controlled system is in the following form [15],

$$\mathbf{G}(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \cdots & \cdots & \cdots & \cdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix} \quad (1)$$

Without loss of generality, let each element of the process transfer function matrix be represented by the first order plus delay time (FOPDT) model and the second order plus delay time (SOPDT) model, both of which can describe most industrial processes.

$$g_{ij}(s) = \frac{k_{ij}}{T_{ij}s + 1} e^{-\tau_{ij}s} \quad (2)$$

$$g_{ij}(s) = \frac{k_{ij}}{a_{0,ij}s^2 + a_{1,ij}s + 1} e^{-\tau_{ij}s} \quad (3)$$

where k is the proportionality coefficient, τ is the delay time, T is the time constant of FOPDT and a_0, a_1 is the first coefficient and second coefficient of FOPDT.

Let $g_{ij}(s) = g_{ij}(j0) \times \bar{g}_{ij}(s)$, where $g_{ij}(j0)$ and $\bar{g}_{ij}(s)$ with $\bar{g}_{ij}(j0) = 1$ are the steady-state gain and the normalized transfer function of $g_{ij}(s)$, respectively. Assuming that the process $\bar{g}_{ij}(s)$ is open-loop stable, the output of which $\bar{y}_i = \bar{g}_{ij}(s)u_j$ initially rests at zero, and applying a unit step disturbance at a process input u_j , we obtain

$$E_{ij} = \int_0^\infty [\bar{y}_i(\infty) - \bar{y}_i(t)]^2 dt \quad (4)$$

which represents the value of energy consumption [8].

For the common FOPDT and SOPDT, the following equations can be derived from Eqs. (2)–(4):

$$E_{ij} = \tau_{ij} + \frac{T_{ij}^2}{2} \quad (5)$$

$$E_{ij} = \tau_{ij} + \frac{a_{1,ij}^2 + a_{0,ij}}{2a_{1,ij}} \quad (6)$$

3.2 The relative energy gain array

For a MIMO system, interaction analysis of the control loops should consider steady-state gain and transient information. The transient character of REGA is measured by the value of energy consumption. Apparently, a small energy consumption value implies a fast dynamic response of the transfer function weak interaction from other loops, whereas a larger value indicates a slow dynamic response and severe coupling with other control loops.

Energy gain $k_{E,ij}$ is defined using the steady-state gain and the energy consumption value in REGA.

$$k_{E,ij} = \frac{g_{ij}(j0)}{E_{ij}} \quad (7)$$

where $g_{ij}(j0)$ is the steady-state gain and E_{ij} is the energy consumption value.

Extending Eq. (7) to all elements of the transfer function matrix $\mathbf{G}(s)$, the energy gain matrix $\mathbf{K}_E$ can be obtained.

$$\mathbf{K}_E = [k_{E,ij}]_{n \times n} = \mathbf{G}(j0) \odot \mathbf{E} \quad (8)$$

where $\odot$ indicates an element-by-element division, $\mathbf{E} = [E_{ij}]_{n \times n}$.

Similar to the definition of RGA, the steady-state gain matrix is replaced by the energy gain matrix $\mathbf{K}_E$ in Eq. (8), and the relative energy gain (REG) between output variable y_i and input variable u_j is defined.

$$\gamma_{ij} = \frac{k_{E,ij}}{k_{E,ij}} \quad (9)$$

where $k'_{E,ij}$ is the energy gain between output variable y_i and input variable u_j when all other loops are closed. Hence, REGA can be calculated by the following equation

$$\mathbf{\Gamma} = \mathbf{K}_E \otimes \mathbf{K}_E^{-T} \quad (10)$$

where $\mathbf{\Gamma} = [\gamma_{ij}]_{n \times n}$.

Similar to RGA based pairing rules, the REGA based loop pairing rules require that manipulated and controlled variables in a process control system should be paired.

(1) Corresponding REGA elements are closed to 1.0.

(2) All paired REGA elements must be positive.

(3) Large REGA elements should be avoided.

Example 1 Continue

REGA can be obtained through Eq. (10).

$$\mathbf{G}_{\text{REGA}} = \begin{bmatrix} 0.6197 & 0.3803 \\ 0.3803 & 0.6197 \end{bmatrix}$$

Reference [8] demonstrates that the diagonal pairing is the best one with the smallest interactions between control loops. The RGA only considers the steady state information so that it can not give a reasonable variable pairing result. Although the ERGA can consider steady-state and transient information, the calculation of ERGA is related to the selection of bandwidth without considering the time delay in frequency domain. In time domain, RGA can not give a reasonable pairing criterion while time delay is very large and AST is constant. However, when the time delay is small and less than the time constant, the loop pairing result from REGA is same to that from RGA. That is to say, the REGA can make up the deficiency of the RGA.

4 INTERACTION DECOMPOSITION PRINCIPLE FOR LARGE-SCALE SYSTEMS

For an $n \times n$ process control system, the control loop interaction can be classified into two extreme situations. (1) When the interaction among control loops is weak, the control system can be designed as n single variable controllers. (2) When the interaction among the control loops is strong, the centralized control scheme is applied to the control system (*i.e.*, only one PID controller). In practical process systems, the controller number must be selected from 1 to n . Considering the interaction, the minimum number of the overall controllers $N_{\min}$ ($N_{\min} \geq 1$) should be obtained for large-scale systems with relatively strong interactions. For others with relatively weak interactions, the control system will contain the maximum controller number $N_{\max}$ ($N_{\max} \leq n$).

For a MIMO system, without interactions in some controlled objects that can be controlled by the conventional single-variable PID controller, it should not be decomposed. However, interaction decomposition should be used for some controlled objects with a strong couple among the control loops. After decomposing the MIMO system, some subsystems can still be controlled by the single-variable PID controller, such as REGA element close to 1, and then several 1×1 control subsystems can be determined. Some other subsystems can be controlled by multivariable PID controllers, such as REGA elements close to 0.5, and then several multivariable control subsystems can be obtained. The following example is used as a demonstration.

Example 2 Consider a 4×4 process with the

following transfer function matrix:

$$\mathbf{G}(s) = \begin{bmatrix} \frac{10e^{-s}}{2s+1} & \frac{3.5e^{-s}}{5s+1} & \frac{e^{-2s}}{16s+1} & \frac{e^{-2s}}{16s+1} \\ \frac{2e^{-s}}{2s+1} & \frac{11e^{-s}}{2s+1} & \frac{0.8e^{-2s}}{16s+1} & \frac{0.9e^{-2s}}{16s+1} \\ \frac{1.8e^{-s}}{2s+1} & \frac{2e^{-s}}{2s+1} & \frac{10e^{-s}}{2s+1} & \frac{20e^{-s}}{6s+1} \\ \frac{4e^{-5s}}{6s+1} & \frac{2e^{-3s}}{10s+1} & \frac{2e^{-s}}{2s+1} & \frac{2.4e^{-s}}{2s+1} \end{bmatrix}$$

REGA can be calculated using Eq. (10):

$$\mathbf{\Gamma} = \begin{bmatrix} 1.0404 & -0.0375 & -0.0059 & 0.0029 \\ -0.0372 & 1.0397 & -0.0024 & -0.0001 \\ -0.0032 & -0.0024 & 0.5539 & 0.4517 \\ -0.0001 & 0.0002 & 0.4544 & 0.5455 \end{bmatrix}$$

In matrix $\mathbf{\Gamma}$, the REGA elements indicate that the control loops y_1-u_1 and y_2-u_2 have weak interaction with other loops, which can be designed into 1×1 subsystems controlled by a single-variable PID controller. Control loops y_3-u_3 and y_4-u_4 present severe couple with other loops, which should not be decomposed, and they form a multivariable control system controlled by a multivariable PID controller. For a large-scale system, the number of controllers is directly related to the design cost and reliability of the control system.

Through the interaction decomposition principle presented in this paper, the upper limit $N_{\max}$ and the lower limit $N_{\min}$ of the controller number can be obtained after analyzing the interaction among the loops and selecting the same threshold value. The controller number is derived from the adjoining matrix, which can be calculated using REGA. According to the second paragraph in this section, the following two definitions of the adjoining matrix can be defined.

Definition 1 The elements in the adjoining matrix $\mathbf{R} = [\gamma_{ij}]_{n \times n}$, which can determine the lower limit $N_{\min}$ of the controller number, should be calculated according to the following criterion

$$\gamma_{ij} = \begin{cases} 0, & |\gamma'_{ij}| < \varepsilon \\ 1, & |\gamma'_{ij}| \geq \varepsilon \text{ or } i = j \end{cases} \quad i, j = 1, 2, \dots, n \quad (11)$$

where γ'_{ij} represents the elements after adjusting the corresponding variables-pairing elements in REGA to the diagonal, and ε is the threshold value.

If $|\gamma'_{ij}| < \varepsilon$, the element value γ'_{ij} is near 0, and the effect of input variable u_j on output variable y_i is so small that it can be ignored. Thus the element of the corresponding place in $\mathbf{R}$ equals 0. If $|\gamma'_{ij}| \geq \varepsilon$, the element value γ'_{ij} is not close to 0, and the effect of input variable u_j on output variable y_i should be

considered. Hence, the element of the corresponding place in $\mathbf{R}$ equals 1.

Definition 2 The elements in the adjoining matrix $\mathbf{Q} = [q_{ij}]_{n \times n}$, which can determine the upper limit $N_{\max}$ of the controller number, should be calculated according to the following criterion

$$q_{ij} = \begin{cases} 0, & \|\gamma'_{ij} - 0.5\| > \varepsilon \\ 1, & \|\gamma'_{ij} - 0.5\| \leq \varepsilon \text{ or } i = j \end{cases} \quad i, j = 1, 2, \dots, n \quad (12)$$

where γ'_{ij} represents the elements after adjusting the corresponding variables pairing-elements in REGA to the diagonal.

If $\|\gamma'_{ij} - 0.5\| > \varepsilon$, the element value γ'_{ij} is not near 0.5, and the effect of input variable u_j on output variable y_i is so small that it can be ignored. Thus the element of the corresponding place in $\mathbf{Q}$ equals 0. If $\|\gamma'_{ij} - 0.5\| \leq \varepsilon$, the element value γ'_{ij} is close to 0.5 and the effect of input variable u_j on output variable y_i is significant so that it must be considered. Hence, the element of the corresponding place in $\mathbf{Q}$ equals 1.

According to Definition 1, if all the elements γ'_{ij} are more than the threshold value, the elements in matrix $\mathbf{R}$ will be 1. Then, the controller number can be decided to be 1 and it is the minimum number of the controller, $N_{\min}$. As the selected threshold value increases, the elements in matrix $\mathbf{R}$, 1, will decrease. Thus the lower limit $N_{\min}$ will increase. As we know, when the elements in REGA are closed to 0.5, the couple among loops is the strongest. According to Definition 2, if $\|\gamma'_{ij} - 0.5\|$ is more than the threshold value, the off-diagonal elements in matrix $\mathbf{Q}$ will be 0. Then, the controller number can be decided to be n . The upper limit $N_{\max}$ can be obtained. As the selected value is large, the value of $N_{\max}$ will decrease. As the selected threshold value increases, the values of $N_{\min}$ and $N_{\max}$ will be equal. Then, the optimal controller number is derived.

Remark 1 The range of the threshold value ε should be selected from 0 to 0.25. If the threshold value is larger than 0.25, 1s in $\mathbf{R}$ will decrease and 1s in $\mathbf{Q}$ will increase, resulting in $N_{\min} > N_{\max}$. Similarly, the range of seeking $N_{\min}$ and $N_{\max}$ will overlap as the

threshold value increases. Fig. 2 shows that the range of γ'_{ij} is selected as the threshold increases. When the threshold value ε is greater than 0.25, the interaction decomposition is the same as that when the threshold value ε is selected from 0 to 0.25. In other words, the value of the upper limit $N_{\max}$ will be smaller than the lower limit $N_{\min}$. Then, the value of the upper limit $N_{\max}$ will be the value of the minimum controller number. The largest threshold value is 0.25.

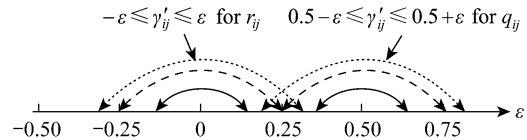


Figure 2 The range representations of value γ'_{ij}

5 ALGORITHM OF INTERACTION DECOMPOSITION

Based on the reasonable variable-pairing determined by REGA, two adjoining matrixes are determined by selecting the same threshold value ε . Then, $N_{\min}$ and $N_{\max}$ can be determined. Finally, the optimal controller number N_{opt} is obtained.

The algorithm for the interaction decomposition of large-scale systems is as follows.

Step 1 Calculate REGA by Eq. (9) and all the paired REGA elements must be positive and closest to 1.0 to obtain the best variable pairing.

Step 2 Exchange the places of the rows or ranks where the selected paired elements locate, such that the elements for the pairing of input and output are along the diagonal.

Step 3 Select the threshold value ε ($0 \leq \varepsilon \leq 0.25$) and determine the adjoining matrixes $\mathbf{R}$ and $\mathbf{Q}$ by Eqs. (10) and (11).

Step 4 Determine $N_{\min}$ and $N_{\max}$ from the adjoining matrix obtained in Step 3. If $N_{\min}$ is not equal to $N_{\max}$, increase ε and go to Step 3.

Step 5 Obtain the optimal controller number N_{opt} .

The following example demonstrates the algorithm of interaction decomposition.

Example 3 Consider a 4×4 process [16] with the following transfer function matrix:

$$\mathbf{G}(s) = \begin{bmatrix} \frac{-9.81e^{-1.59s}}{11.36s + 1} & \frac{0.374e^{-7.75s}}{22.22s + 1} & \frac{-2.368e^{-27.33s}}{33.3s + 1} & \frac{-11.3e^{-3.79s}}{(21.74s + 1)^2} \\ \frac{5.984e^{-2.24s}}{14.29s + 1} & \frac{-1.986e^{-0.71s}}{66.67s + 1} & \frac{0.422e^{-8.72s}}{(250s + 1)^2} & \frac{5.24e^{-60s}}{400s + 1} \\ \frac{2.38e^{-0.42s}}{(1.43s + 1)^2} & \frac{0.0204e^{-0.59s}}{(7.14s + 1)^2} & \frac{0.513e^{-s}}{s + 1} & \frac{-0.33e^{-0.68s}}{(2.38s + 1)^2} \\ \frac{-11.3e^{-3.79s}}{(21.74s + 1)^2} & \frac{-0.176e^{-0.48s}}{(6.9s + 1)^2} & \frac{15.54e^{-s}}{s + 1} & \frac{4.48e^{-0.52s}}{11.11s + 1} \end{bmatrix}.$$

Calculating REGA and re-arranging the places for the pairing of input and output along the diagonal, the following matrixes are obtained:

$$\Gamma = \begin{bmatrix} 0.3094 & -0.0639 & -0.0072 & 0.7618 \\ -0.0294 & 1.0456 & 0.0000 & -0.0163 \\ 0.7116 & 0.0139 & 0.0636 & 0.2109 \\ 0.0084 & 0.0044 & 0.9436 & 0.0436 \end{bmatrix}$$

$$\Gamma' = \begin{bmatrix} 0.7618 & -0.0639 & 0.3094 & -0.0072 \\ -0.0163 & 1.0456 & -0.0294 & 0.0000 \\ 0.2109 & 0.0139 & 0.7116 & 0.0636 \\ 0.0436 & 0.0044 & 0.0084 & 0.9436 \end{bmatrix}.$$

Selecting the threshold values $\varepsilon = 0.05, 0.1, 0.15$ and 0.2 , the following adjoining matrixes are obtained, respectively,

$$R_1 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$R_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$R_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$R_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Q_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

A series of controller numbers from the above matrixes can be determined. Fig. 3 shows the tendency of the controller number as the threshold value increases.

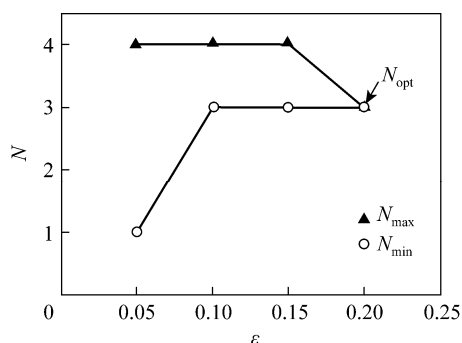


Figure 3 The tendency of controller numbers

Figure 3 shows three kinds of decomposition results for Example 3: only one controller (centralized control structure), four controllers (decentralized control structure) and three controllers (block decentralized control structure). The decentralized control structure is the pairing $\{(y_1-u_4), (y_2-u_2), (y_3-u_1), (y_4-u_3)\}$, and the block decentralized control structure is $\{(y_1, y_3)-(u_4, u_1), (y_2-u_2), (y_4-u_3)\}$. Applying with the RGA and ERGA and decomposing the control system, we can get the control structure that is $\{(y_1-u_4), (y_2-u_2), (y_3-u_1), (y_4-u_3)\}$. Thus the number of optimal controller derived by RGA and ERGA is 4. The result of decomposing the control system through the RGA is the same to REGA. To compare the control performance of these three control structures, PID controllers are designed based on the internal model control (IMC) controller tuning rules [17, 18]. Fig. 4 compares the simulation results.

The control structure of the three PID controllers is optimal from the results of the simulation. The control structure of only one controller ignores the inner interaction of the system, and it is very difficult to tune its controller parameters. The control structure of the four subsystems controlled by four PID controllers does not consider the strong interaction among the control loops. We can say that the RGA and ERGA can not give a reasonable loop pairing criterion and the correct decomposing result. Hence, the use of the loop pairing method REGA and the interaction decomposition principle of large-scale system to achieve perfect control performance are significant. Example 3 demonstrates the effectiveness and necessity of the interaction analysis and interaction decomposition.

6 CASE STUDY: TENNESSEE EASTMAN PROCESS

The Tennessee Eastman (TE) plant-wide control system, developed by Downs and Vogel [19], has served as a useful vehicle for testing control strategies developed by different investigators. The authors have put multivariable control, on-line optimization, predictive control, identification and process diagnostics as potential applications. The TE process consists of five major unit operations: reactor, product condenser, vapor-liquid separator, recycle compressor and product stripper. The details of the process can be found in Ref. [19].

The TE process involves 41 measurements and 12 manipulated variables. From the design experience, eight flow and two temperature cascade loops can be closed [20]. The flow loops include four feed streams: purge stream, stripper bottoms, separator bottoms and stripper steam flow. The two temperature cascades include the condenser and reactor cooling streams, as analyzers are typically less reliable than the more common temperature, pressure, flow, and level sensors. Analyzer loops are also typically slower. Thus 19 analyzer measurements are eliminated, then there are $41 - 10 - 19 = 12$ potential variable to be controlled.

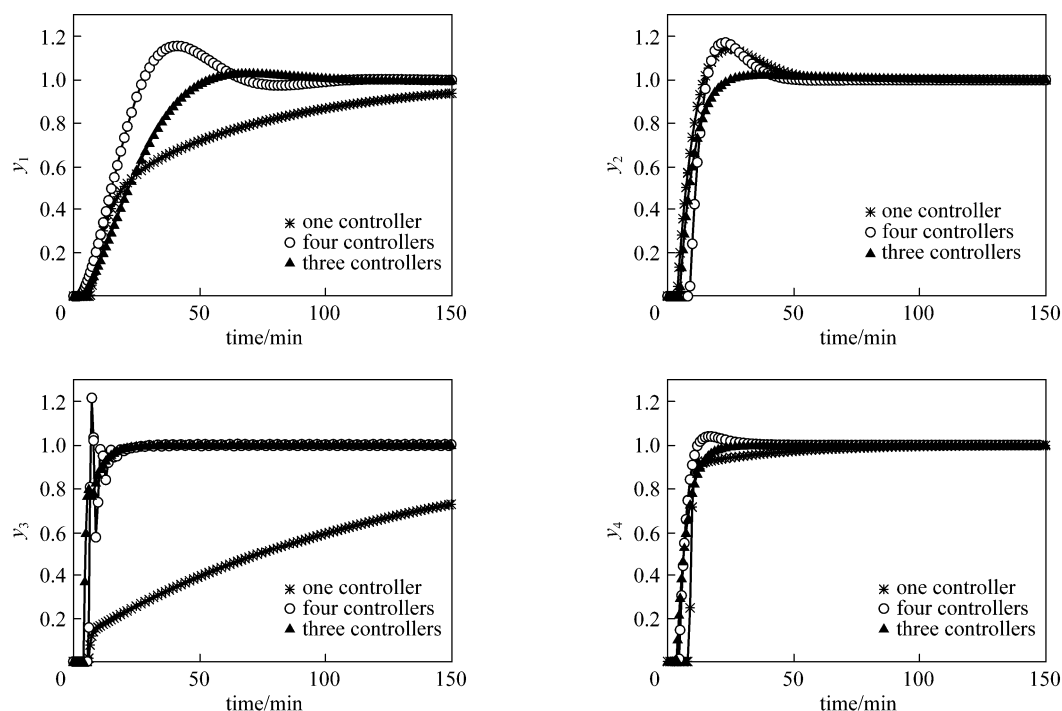


Figure 4 The results of simulation comparison for Example 3

Table 1 Manipulated variables and controlled variables

Manipulated variables	Variable numbers	Controlled variables	Variable numbers
D feed flow	u_1	recycle flow	y_1
E feed flow	u_2	reactor feed rate	y_2
A feed flow	u_3	reactor pressure	y_3
C feed flow	u_4	reactor level	y_4
compressor recycle valve	u_5	reactor temperature	y_5
purge valve	u_6	product separator temperature	y_6
separator pot liquid flow	u_7	product separator level	y_7
stripper liquid product flow	u_8	product separator pressure	y_8
stripper steam valve	u_9	stripper pressure	y_9
reactor cooling water flow	u_{10}	stripper underflow	y_{10}
condenser cooling water flow	u_{11}	stripper temperature	y_{11}
agitator speed	u_{12}	compressor work	y_{12}

Those controlled and manipulated variables are listed in Table 1. Since the dynamic model of the TE process is available, the transfer function matrixes of the process are identified using the Matlab identification toolbox [21, 22]. The identification results are listed in Appendix Table A1.

In practical chemical processes, the derivation of subsystems is judged by the physical location. The TE process can be decomposed into three subsystems, reaction system, separation system and stripper system [23]. The decomposition result is in accord with engineers'

experience. Fig. 5 shows the three subsystems for the TE process.

Although some proposed techniques may satisfy control structure selection, no systematic and theoretic method is applicable to the control structure selection. The interaction analysis and interaction decomposition proposed in this paper can make up the shortage in the control structure selection. Calculating REGA and re-arranging the places for the pairing of input and output along the diagonal, the following matrixes are obtained:

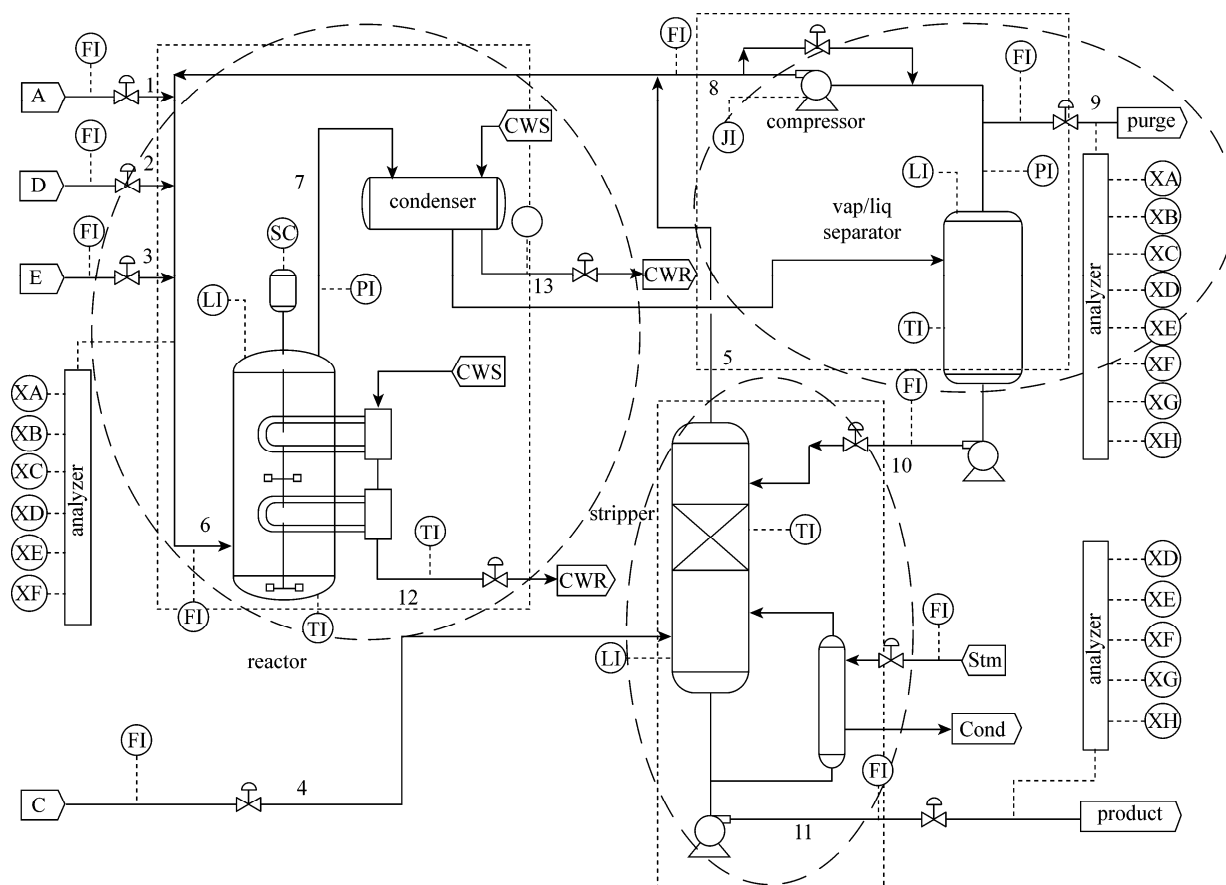


Figure 5 The result of decomposition for TE process

1, A—feed A; 2, D—feed D; 3, E—feed E; 4, C—feed C; 5—stripper overhead; 6—reactor feed; 7—reactor product; 8—recycle; 9—purge; 10—separation liquid; 11—product; CWS—cooling water supply; CWR—cooling water recycle; vap—vapour; liq—liquid; Stm—stream; Cond—condensate water; XA–XH—composition of A–H; FI—flow rate indicator; TI—temperature indicator; LI—level indicator; PI—pressure indicator; SC—safty controller

$$\Gamma = \begin{bmatrix}
 0.4867 & -0.3961 & 0.4544 & -0.2035 & 0.119 & -0.3713 & -0.1297 & 0.0149 & 0.0049 & 0.1418 & 0.1273 & 1.5223 \\
 0.0477 & 0.6899 & -0.5162 & 0.0412 & 0.0366 & 0.0585 & -0.2000 & 0.0585 & 0.0022 & 0.3441 & -0.0542 & -0.0083 \\
 0.3669 & 0.0068 & 2.1296 & -0.0466 & 0.0786 & -1.4689 & 0.0937 & -0.0233 & -0.0415 & 0.1591 & 0.2009 & -0.1152 \\
 1.1598 & -0.0484 & -0.0027 & -0.0070 & 0.1306 & -0.0008 & -0.5038 & 0.0198 & 0.0012 & 0.0213 & 0.2166 & 0.3834 \\
 -0.0061 & 0.2241 & 0.0247 & 0.0595 & 0.0011 & -0.0229 & 0.0071 & -0.0625 & -0.004 & 0.2804 & -0.0098 & 0.3566 \\
 0.0063 & -0.1976 & -0.1244 & 0.1323 & 0.0172 & 0.2774 & -0.0266 & 0.0264 & -0.0034 & 0.1140 & 1.1322 & -0.0240 \\
 -0.1867 & 0.0085 & 0.0001 & 0.0024 & 0.0023 & -0.0011 & 1.6893 & -0.0254 & -0.0453 & 0.0043 & -0.0075 & -0.1409 \\
 -0.0434 & -0.0112 & -0.1294 & -0.8065 & -0.1084 & 2.7777 & -0.2002 & 0.0814 & 0.0249 & -0.2268 & -0.0001 & 0.0120 \\
 0.681 & -0.0092 & -0.1023 & 1.785 & 0.0233 & -0.3298 & 0.2137 & -0.1174 & 0.0102 & -0.0198 & -0.0081 & -0.2536 \\
 0.3823 & 0.0009 & -0.0001 & -0.0036 & -0.0647 & 0.0027 & -0.7099 & 1.2016 & 0.0093 & -0.0007 & -0.0109 & 0.1931 \\
 0.0009 & -0.0098 & -0.0004 & 0.0078 & 0.0002 & -0.0123 & -0.0206 & -0.0051 & 1.0626 & 0.0155 & -0.0386 & -0.0002 \\
 0.0527 & 0.2371 & 0.0279 & 0.0391 & 0.8714 & 0.0909 & 0.7871 & -0.1688 & -0.0210 & 0.1666 & -0.5478 & -0.1952
 \end{bmatrix}$$

$$\Gamma' = \begin{bmatrix} 1.5223 & -0.3961 & -0.4544 & 0.4867 & 0.2418 & 0.1273 & -0.1297 & -0.1313 & -0.1235 & 0.0149 & 0.0049 & 0.0119 \\ -0.0083 & 0.6899 & -0.5162 & 0.0477 & 0.3441 & -0.0542 & -0.2000 & 0.0585 & 0.0412 & 0.0585 & 0.0022 & 0.0366 \\ -0.1152 & 0.0068 & 2.1296 & 0.3669 & 0.1591 & 0.2009 & 0.0937 & -0.1489 & -0.0466 & -0.0233 & -0.0415 & 0.0786 \\ 0.3834 & -0.0484 & -0.0027 & 1.1598 & 0.0213 & 0.2166 & -0.5038 & -0.0008 & -0.0070 & 0.0198 & 0.0012 & 0.1306 \\ -0.3566 & 0.2241 & 0.0247 & -0.0061 & 0.2804 & -0.0098 & 0.0071 & -0.0229 & 0.0595 & -0.0625 & -0.0040 & 0.0011 \\ -0.0240 & -0.1976 & -0.1244 & 0.0063 & 0.1140 & 1.1322 & -0.4266 & 0.2744 & 0.1323 & 0.0264 & -0.0034 & 0.4172 \\ -0.1409 & 0.0085 & 0.0001 & -0.1867 & 0.0043 & -0.0075 & 1.6893 & -0.3011 & 0.0024 & -0.0254 & -0.0543 & 0.0023 \\ 0.0120 & -0.0122 & 0.1294 & -0.0434 & -0.2268 & -0.3601 & -0.2002 & 2.7777 & -0.8065 & 0.0814 & 0.0249 & -0.2884 \\ -0.2536 & -0.0092 & -0.1023 & 0.0681 & -0.0198 & -0.0081 & 0.2137 & -0.3298 & 1.7850 & -0.3874 & 0.0102 & 0.0233 \\ 0.1931 & 0.0009 & -0.0001 & 0.3823 & -0.0007 & -0.0109 & -0.1099 & 0.0027 & -0.0036 & 1.2016 & 0.6893 & -0.0647 \\ -0.0002 & -0.0098 & -0.0004 & 0.0009 & 0.0155 & -0.0386 & -0.0206 & -0.0123 & 0.0078 & -0.0051 & 1.0626 & 0.0002 \\ -0.1952 & 0.2371 & 0.0279 & 0.0527 & 0.1666 & -0.5478 & 0.0871 & 0.0909 & 0.0391 & -0.1688 & -0.2010 & 0.8714 \end{bmatrix}$$

Selecting the threshold values $\varepsilon = 0.002, 0.05, 0.1, 0.15, 0.2$ and 0.23 , the following adjoining matrixes are determined, respectively,

$$R_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$R_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$R_6 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$Q_6 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The number of controllers for the TE process can be determined by calculating these matrixes. Fig. 6 shows the tendency of the controller number as the threshold value increases.

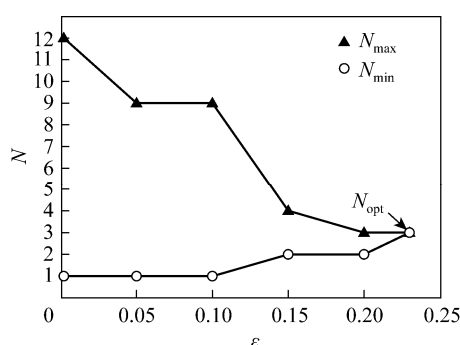


Figure 6 The tendency of controller numbers for the TE process

With the interaction decomposition principle applied to the TE process, adjoining matrixes R_6 and Q_6 indicate that the optimal controller number is 3, when $N_{\max} = N_{\min}$, indicating that the TE plant can be decomposed into three subsystems, namely, $(y_1, y_2, y_3, y_4,$

$y_5)-(u_{12}, u_2, u_3, u_1, u_{10}), (y_6, y_7, y_8, y_{12})-(u_{11}, u_7, u_6, u_5)$ and $(y_9, y_{10}, y_{11})-(u_4, u_8, u_9)$. Result of the decomposition shows that the sites of the three subsystems are consistent with the physical place of the TE plant, according to the decomposition of the practical control system design. Result of the interaction decomposition for the TE plant demonstrates that the interaction decomposition proposed in the present paper is useful in practice and it can server as a guideline for practical control structure design. Fig. 5 shows the results of the interaction decomposition and the practical decomposition for the TE process.

7 CONCLUSIONS

The paper aims to determine the optimal controller number by decomposing large-scale systems. From the perspective of control system design, a new variable pairing is introduced to consider both steady-state and transient information, and the interaction decomposition principle for the large-scale systems is proposed. By selecting the threshold value, REGA is transferred into two adjoining matrixes, which can measure the interaction relationship between the input variables and output variables, and determine the upper limit $N_{\max}$ and the lower limit $N_{\min}$. As the threshold value increases, the value $N_{\max}$ will equal $N_{\min}$. At this point, the optimal controller number N_{opt} is determined. By analyzing the TE plant, the validity and feasibility of interaction decomposition are illustrated.

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APPENDIX

Table A1 Some transfer function matrixes of the TE process

Item	u_1	u_2	u_3	u_4	u_5	u_6
y_1	$\frac{-0.0028e^{-5.4s}}{43.07s + 1}$	$\frac{0.5e^{-5.39s}}{23.59s + 1}$	$\frac{-11.2025e^{-5.35s}}{38.39s + 1}$	$\frac{1.65e^{-5.4s}}{23.06s + 1}$	$\frac{-0.0187e^{-4.63s}}{21.05s + 1}$	$\frac{-7.0837e^{-5.37s}}{44.94s + 1}$
y_2	$\frac{-0.003e^{-4.6s}}{34.84s + 1}$	$\frac{0.78e^{-3.9s}}{25.59s + 1}$	$\frac{-8.2062e^{-5.7s}}{41.77s + 1}$	$\frac{3.2054e^{-7.9s}}{35s + 1}$	$\frac{-0.036e^{-4.66s}}{25.4s + 1}$	$\frac{-7.682e^{-8.96s}}{38.32s + 1}$
y_3	$\frac{-0.2796e^{-11.43s}}{55.6s + 1}$	$\frac{0.4991e^{-11.44s}}{58.48s + 1}$	$\frac{-3103.25e^{-15.44s}}{54.42s + 1}$	$\frac{287.05e^{-10.42s}}{99.82s + 1}$	$\frac{7.78e^{-7.32s}}{42.93s + 1}$	$\frac{-965.7e^{-11.45s}}{73.91s + 1}$
y_4	$\frac{1.28e^{-6.83s}}{39.53s + 1}$	$\frac{1.5451e^{-6.84s}}{85.53s + 1}$	$\frac{2.53e^{-10.07s}}{63.65s + 1}$	$\frac{1.63e^{-6.84s}}{49.04s + 1}$	$\frac{3.36e^{-7.46s}}{27.49s + 1}$	$\frac{-0.302e^{-9.44s}}{45.6s + 1}$
y_5	$\frac{0.0022e^{-4.9s}}{75.35s + 1}$	$\frac{2.24e^{-7.9s}}{65.9s + 1}$	$\frac{6.843e^{-7.73s}}{83.2s + 1}$	$\frac{1.662e^{-6.46s}}{41.59s + 1}$	$\frac{0.0141e^{-34.64s}}{81.36s + 1}$	$\frac{0.8968e^{-5.65s}}{37.54s + 1}$
y_6	$\frac{0.0109e^{-12.78s}}{48.034s + 1}$	$\frac{1.52e^{-3.6s}}{24.59s + 1}$	$\frac{67.8248e^{-13.58s}}{30.87s + 1}$	$\frac{-7.4203e^{-13.71s}}{56.3s + 1}$	$\frac{-0.1954e^{-27.14s}}{42.21s + 1}$	$\frac{21.0053e^{-3.57s}}{26.18s + 1}$
y_7	$\frac{0.8e^{-4.6s}}{12.94s + 1}$	$\frac{0.93e^{-4.6s}}{15.64s + 1}$	$\frac{2.04e^{-8.25s}}{40.8s + 1}$	$\frac{0.82e^{-4.6s}}{22.51s + 1}$	$\frac{2.26e^{-8.25s}}{21.5s + 1}$	$\frac{1.25e^{-6.4s}}{14.09s + 1}$
y_8	$\frac{-0.2744e^{-16.7s}}{46.07s + 1}$	$\frac{0.485e^{-9s}}{33.29s + 1}$	$\frac{-3053.87e^{-24.95s}}{53.13s + 1}$	$\frac{281.139e^{-25.2s}}{60.54s + 1}$	$\frac{8.54e^{-17.97s}}{48.74s + 1}$	$\frac{-948.7e^{-17.76s}}{58.73s + 1}$
y_9	$\frac{-0.308e^{-11.4s}}{37.02s + 1}$	$\frac{0.56.56e^{-9.38s}}{26.44s + 1}$	$\frac{-3377.3005e^{-25.88s}}{98.29s + 1}$	$\frac{319.99e^{-16.3s}}{42.87s + 1}$	$\frac{3.5e^{-16.8s}}{56.83s + 1}$	$\frac{-1059.65e^{-23.24s}}{79.72s + 1}$
y_{10}	$\frac{0.36e^{-12.11s}}{33.09s + 1}$	$\frac{0.42e^{-12s}}{38.53s + 1}$	$\frac{0.91e^{-7.76s}}{13.91s + 1}$	$\frac{0.37e^{-12.11s}}{23.15s + 1}$	$\frac{e^{-7.75s}}{15.46s + 1}$	$\frac{0.58e^{-12.78s}}{23.66s + 1}$
y_{11}	$\frac{0.0095e^{-12.22s}}{69.48s + 1}$	$\frac{1.214e^{-12.06s}}{88.68s + 1}$	$\frac{2.63e^{-11.62s}}{55.12s + 1}$	$\frac{-6.4259e^{-12.25s}}{37.47s + 1}$	$\frac{-0.038e^{-17.6s}}{40.18s + 1}$	$\frac{20.112e^{-16.3s}}{45.29s + 1}$
y_{12}	$\frac{-0.0159e^{-19.6s}}{38.79s + 1}$	$\frac{6.35e^{-18.9s}}{61.63s + 1}$	$\frac{13.75e^{-13.4s}}{57.08s + 1}$	$\frac{5.55e^{-12.33s}}{68.22s + 1}$	$\frac{12.97e^{-3.54s}}{37.41s + 1}$	$\frac{-27.43e^{-12.37s}}{48.53s + 1}$

Table A1 (Continued)

Item	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}
y_1	$\frac{0.264e^{-25.37s}}{86.56s+1}$	$\frac{0.58e^{-8.97s}}{70.6s+1}$	$\frac{0.004e^{-5s}}{69s+1}$	$\frac{-0.651e^{-8.94s}}{79.36s+1}$	$\frac{1.28e^{-13.5s}}{43.58s+1}$	$\frac{0.1256e^{-2.97s}}{34.7s+1}$
y_2	$\frac{0.2346e^{-11.97s}}{47.56s+1}$	$\frac{0.91e^{-9.9s}}{79.4s+1}$	$\frac{0.0009e^{-19.9s}}{58s+1}$	$\frac{-0.6652e^{-5.9s}}{35.4s+1}$	$\frac{2.314e^{-6.5s}}{45.74s+1}$	$\frac{0.1291e^{-16.5s}}{46.2s+1}$
y_3	$\frac{11.968e^{-11.44s}}{83.99s+1}$	$\frac{58.022e^{-19s}}{64.32s+1}$	$\frac{0.048e^{-10.4s}}{29.48s+1}$	$\frac{-31.4794e^{-19.1s}}{63.9s+1}$	$\frac{-52.91e^{-12.5s}}{63.1s+1}$	$\frac{6.1135e^{-19.1s}}{48.3s+1}$
y_4	$\frac{1.97e^{-13.5s}}{67.3s+1}$	$\frac{1.6e^{-8.94s}}{80.72s+1}$	$\frac{0.0003e^{-13.5s}}{33.96s+1}$	$\frac{0.368e^{-9.42s}}{24.29s+1}$	$\frac{4.8603e^{-6.837s}}{36.4s+1}$	$\frac{0.023e^{-19.3s}}{47.5s+1}$
y_5	$\frac{0.0281e^{-17.73s}}{84.9s+1}$	$\frac{2.55e^{-9.85s}}{32.96s+1}$	$\frac{0.0005e^{-13.13s}}{48.7s+1}$	$\frac{1.1106e^{-2.6s}}{14.29s+1}$	$\frac{0.098e^{-13.07s}}{33.05s+1}$	$\frac{-0.2149e^{-20.4s}}{56.4s+1}$
y_6	$\frac{0.285e^{-3.59s}}{29.57s+1}$	$\frac{1.72e^{-5.96s}}{22.66s+1}$	$\frac{-0.002e^{-4.5s}}{20.7s+1}$	$\frac{2.2502e^{-14.9s}}{47.09s+1}$	$\frac{4.47e^{-3.6s}}{18.6s+1}$	$\frac{-0.4359e^{-16.01s}}{45.79s+1}$
y_7	$\frac{221.31e^{-2.4s}}{20.5s+1}$	$\frac{1.07e^{-4.6s}}{20.45s+1}$	$\frac{0.0563e^{-9.67s}}{41.99s+1}$	$\frac{1.2122e^{-16.86s}}{22.83s+1}$	$\frac{2.7634e^{-13.7s}}{26.8s+1}$	$\frac{1.0019e^{-7.35s}}{28.61s+1}$
y_8	$\frac{11.8e^{-8.3s}}{48.17s+1}$	$\frac{58.531e^{-3.2s}}{49.02s+1}$	$\frac{0.0465e^{-13s}}{39.7s+1}$	$\frac{-30.8414e^{-16.1s}}{61.06s+1}$	$\frac{-0.009e^{-12.25s}}{48.1s+1}$	$\frac{6.0093e^{-14.1s}}{33.18s+1}$
y_9	$\frac{12.89e^{-15.4s}}{45.05s+1}$	$\frac{66.08e^{-7.92s}}{55.2s+1}$	$\frac{0.0561e^{-12.02s}}{48.4s+1}$	$\frac{-34.4602e^{-33.5s}}{81.2s+1}$	$\frac{21.93e^{-9.6s}}{37.9s+1}$	$\frac{6.7e^{-9.03s}}{62.78s+1}$
y_{10}	$\frac{3.08e^{-12.79s}}{44.96s+1}$	$\frac{10.49e^{-2.31s}}{21.64s+1}$	$\frac{0.048e^{-20.1s}}{33.4s+1}$	$\frac{0.0617e^{-21.31s}}{72.96s+1}$	$\frac{1.2301e^{-12.9s}}{31.6s+1}$	$\frac{0.4655e^{-16.4s}}{37.6s+1}$
y_{11}	$\frac{0.413e^{-15.64s}}{46.45s+1}$	$\frac{1.4e^{-20.7s}}{41.92s+1}$	$\frac{0.0368e^{-1.83s}}{20.7s+1}$	$\frac{1.87e^{-18.4s}}{44.5s+1}$	$\frac{-4.12e^{-9.8s}}{62.7s+1}$	$\frac{-0.3615e^{-8.4s}}{56.35s+1}$
y_{12}	$\frac{2.5e^{-19s}}{43.9s+1}$	$\frac{7.37e^{-17.2s}}{58.99s+1}$	$\frac{0.009e^{-15s}}{62s+1}$	$\frac{-4.51e^{-18.05s}}{35.13s+1}$	$\frac{19.096e^{-15s}}{48.5s+1}$	$\frac{0.8732e^{-19.3s}}{56.6s+1}$